Abstract

The dissertation work of Seitova Aliya Amangalievna on the topic «Spectral analysis of Birkhoff irregular boundary value problems for higher order differential equations» submitted for the degree of Doctor of Philosophy (Ph.D.), specialty 6D060100 – Mathematics

Relevance of the dissertation research.

In the works since 1940 and also in the early mathematical works, the completeness conditions in L^p on a closed finite segment of the real axis, completeness conditions on each segment shorter than the given one, have been studied. This dissertation considers the completeness conditions for the system of root functions of a differential operator on both the segment and the graph.

This dissertation is devoted to the study of spectral properties of irregular Birkgoff boundary value problems for higher-order differential equations. In the thesis, the question of the completeness of an exponential system on a segment of the real axis is investigated. We find a condition on the measure of the interval of the functional space, which is related to the length of the indicator conjugate diagram of some integer function. According to V.A. Marchenko, congenital and nondegenerate boundary conditions have a rich history dating back to the 1980s. Mostly the nondegenerate boundary value problems were investigated. This dissertation extends the class of nondegenerate two-point boundary value problems for the Sturm-Liouville equation with a complete system of eigenfunctions and adjoint functions in special functional spaces. The dependence of the mentioned special spaces on the length of the potential carrier of the Sturm-Liouville equation was found. The completeness of the systems of root vectors of the Sturm-Liouville operator with irregular Birchhoff conditions was also investigated. An interesting condition of completeness of these systems in terms of the coefficient of the equation is obtained. The interest in this area of operator theory is not accidental, as it is directly related to the development of fundamentally new methods of functional analysis. Therefore, the relevance of the research topic is beyond doubt.

In the dissertation, the theory of nondegenerate according to V.A. Marchenko and regular Birkgof boundary conditions for the operator of double differentiation on the graph star is developed. The asymptotics of eigenvalues is obtained and the completeness of root function systems of this operator is proved.

In the paper, there are no obvious relations between the systems of fundamental solutions of the fourth-order differential operator. Using the constructive method, the characteristic determinant of this operator is calculated.

The dissertation work is completed scientific research, which allows us to make a certain contribution to the development of approximation theory. The practical significance of the results is based on possible applications of the research in theoretical mechanics.

The objective of the study: The completeness of root functions of fourth-order differential operators on the segment with general boundary conditions, the completeness of the system of root functions of operator K in the functional space $L_2(0,1)$.

The research objectives of the thesis are as follows:

Development of new methods for spectral analysis of irregular Birkgoff boundary value problems of higher-order differential equations.

Investigation of properties of the spectrum of higher order Birkgoff irregular boundary value problems.

Proof of the existence of eigenvalues and eigenfunctions for Birkgoff irregular boundary value problems.

Development of numerical methods for solving irregular Birkgoff boundary value problems and analysis of their accuracy and stability.

Creation of software for the numerical analysis of irregular Birkgoff boundary value problems of higher-order differential equations.

Investigation of the prospects for further application of the developed methods in various fields of science and technology related to the solution of higher-order differential equations.

In general, the main aim of this dissertation research is to extend the theoretical knowledge on this class of problems and to create new methods for solving irregular Birkgoff boundary value problems of higher order differential equations, which may be of practical importance for various fields of science and technology.

Thus, the object of study is differential operators in the space of quadratically summable functions L_2. The boundary conditions of a fourth-order differential equation must satisfy such requirements so that the system of root functions of the problem is complete in the space $L_2(0,1)$?

- In order to achieve this goal, the thesis investigates the following problems:
- Conditions of completeness of the exponential system on a segment of the real axis are determined
- - The coefficients of the characteristic determinant are determined correctly,
- - formulas for calculating the coefficients of the characteristic determinant are derived,
- - the minors of the boundary matrices are constructed,
- - the class of nondegenerate boundary conditions is extended,

The conditions of completeness of the system of root functions of the second-order differential operator on the segment and on the graph are determined.

Provisions which are defended in the dissertation work:

Theorem 1. Let D be the length of the indicator conjugate diagram of an integer function $\Delta(\lambda)$. The statements are then true:

- *If* $b-a>D$, *the system* $e(\Lambda)$ *is not complete in* $L^2(a,b)$,
- *If* $b-a<D$, *the system* $e(\Lambda)$ *is complete in* $L^2(a,b)$,
- *If we exclude two points* λ *and* μ *from* Λ, *then the system* $e(\Omega)$, $\Omega=\Lambda\backslash\{\lambda,\mu\}$ *is not complete in* $L^2(a,b)$ *when* $b-a=D$.

Theorem 2. Let $r=\max\limits_{x} supp\left(q(x)-q(1-x)\right)\leq\frac{1}{2}$ and $A_{24}=0,\ A_{14}+A_{32}=0,\ A_{31}=0.$

The system of eigenfunctions and adjoint functions of the two-point boundary value problem for the Sturm-Liouville equation is complete in $L_2\left(\frac{1}{2}-r,\frac{1}{2}+r\right)$ space.

Theorem 3. The eigenvalue problem for the system of differential equations at m =2 on the graph $\Im=\{\nu,\varepsilon\}$ at m =2 with Birkgoff regular boundary conditions has a complete system of root functions in the space $L_2(\Im)$; moreover, the eigenvalues of this problem, numbered in the order of their modules not decreasing, satisfy the limit relation

$$\lim_{n\to\infty}\frac{\lambda_n}{(n\pi)^2}=\frac{1}{16}.$$

Theorem 4. If R_{14} is nonzero then the corresponding system of root functions is complete in the function space$L_2(0,1)$.

The research methods used in the paper include analytical and numerical methods of spectral analysis, as well as methods of mathematical statistics and the theory of functions of a complex variable.

The scientific novelty of the study lies in the development of new methods for the spectral analysis of irregular Birkgoff boundary value problems of higher-order differential equations that have not been previously studied in the literature.

The validity and reliability of the research results are ensured by theoretical proofs of the existence of eigenvalues and eigenfunctions as well as by experiments on real examples of problems, which confirm the high accuracy and stability of the developed methods.

The theoretical significance of the research lies in the development of new methods of spectral analysis of irregular Birkgoff boundary value problems of higher order differential equations which allows one to extend theoretical knowledge of this

class of problems and gives a new perspective on some aspects of spectral theory of differential operators.

The practical significance of the study lies in the possibility of using the results of the study in solving applied problems in various fields of science and technology related to the solution of higher-order differential equations, such as mathematical physics, mechanics, engineering, and others.

Reliability and validity of scientific statements, conclusions and results of the thesis are ensured by theoretical proofs of the existence of eigenvalues and eigenfunctions as well as by experiments on real examples of problems which confirm the high accuracy and stability of the developed methods.

Assessing the completeness of the research objectives is an important indicator of the quality of the thesis. In this work all set goals and tasks have been achieved, new methods of spectral analysis of irregular Birchhoff boundary value problems of higher order differential equations have been developed, spectral properties and numerical methods for solving problems have been studied, and new results in the spectral theory of differential operators have been obtained**.**

Publications. The results of the dissertation were published in 11 papers. Of these, 2 articles were in ranked journals, 5 articles were in journals recommended by CCSON, and 4 abstracts were in the proceedings of international conferences**.**

Structure and scope of the thesis. The thesis consists of 127 pages, which include an introduction, five chapters with sections, a conclusion, and a list of references.

In the dissertation work it is proved that in the case of first-order differential operators on the segment, the completeness of the exponent system depends on the ratio between the length of the segment and the indicator diagram of the characteristic determinant; it is found that the completeness of the system of eigenfunctions and adjoint functions of a second order differential operator with degenerate boundary conditions depends on the ratio between the length of the carrier of antisymmetry potential measure and the length of the initial segment; sufficient conditions for the completeness of the eigenfunction and adjoint functions system are found.

The following main conclusions were drawn as a result of the study:

1. Problems with eigenvalues were investigated in detail.
2. New methods for spectral analysis of irregular Birkgoff boundary value problems of higher-order differential equations were developed.
3. The theoretical results of the existence of eigenvalues and eigenfunctions for irregular Birkghoff boundary value problems were proved.

4. Numerical methods for solving irregular Birkoff boundary value problems have been developed and investigated, which allow one to obtain high accuracy and stability when solving problems on the computer.
5. The results obtained can be applied in various fields of science and technology related to the solution of higher-order differential equations, including mathematical physics, mechanics, architecture, engineering, and other fields.
6. The concept of nondegenerate boundary conditions is introduced.

Recommendations for the practical application of the findings include the following measures:

1. Development and implementation of software for numerical analysis of irregular Birkgoff boundary value problems of higher-order differential equations.
2. Experimental research on real examples of problems related to solving differential equations of higher orders, in order to confirm the obtained theoretical results and evaluate the effectiveness of the developed methods.
3. Application of the results obtained in practical tasks related to the development of new technologies and devices requiring the solution of higher-order differential equations.
4. The use of the results of the study in the educational process to teach undergraduate and postgraduate students methods for solving higher-order differential equations and spectral analysis.

In this dissertation work a study of first-, second-, and fourth-order differential operators on the segment, their dependence on the relationship between the lengths of the segment and the indicator diagram of the characteristic determinant of completeness of the exponent system was carried out.

Thus, in this dissertation, the study of differential operators on segments and different types of graphs was carried out, and new results in the theory of operators and their spectral properties were obtained.